

6.8 - Indeterminate Forms and l'Hospital's Rule

Preparation (Watch at Home)

6.8a

Not indeterminate

$$\frac{1}{0} \rightarrow \lim_{x \rightarrow 0} \frac{1}{x} = \infty$$

$$\frac{0}{0}, \frac{\infty}{\infty}, 0^0, \infty^0, 0 \cdot \infty, \infty - \infty, 1^\infty$$

$$\frac{0}{0} \rightarrow 0$$

$$\frac{\infty}{\infty} \rightarrow \infty$$

$$0^\infty = 0$$

$$1, 2, 3, \dots$$

$$\frac{p}{q}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}$$

$$0, \dots$$

l'Hôpital's Rule for indeterminate forms

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \rightarrow a} f(x) = 0 \text{ and } \lim_{x \rightarrow a} g(x) = 0$$

$$\text{or that } \lim_{x \rightarrow a} f(x) = \pm\infty \text{ and } \lim_{x \rightarrow a} g(x) = \pm\infty$$

$$\text{Then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \text{ if the limit on the right-hand side exists or is } \infty \text{ or } -\infty$$

Indeterminate forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, \infty^0, \text{ and } 1^\infty$$

Exercises:

Find the limit. Use l'Hôpital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hôpital's Rule doesn't apply, explain why.

6.8b

$$\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x}$$

$$\lim_{\theta \rightarrow \pi} \frac{1 + \cos \theta}{1 - \cos \theta}$$

$$\lim_{x \rightarrow \infty} \frac{\ln \sqrt{x}}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\sinh x - x}{x^3}$$

6.8d

$$\lim_{x \rightarrow -\infty} x \ln \left(1 - \frac{1}{x} \right)$$

$$\lim_{x \rightarrow 1^+} [\ln(x^7 - 1) - \ln(x^5 - 1)]$$

6.8e

$$\lim_{x \rightarrow 0^+} (\tan 2x)^x$$

L'Hospital's Rule :

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$,

or $x \rightarrow \infty$

$$\text{then } \lim_{\substack{x \rightarrow a \\ x \rightarrow \infty}} \frac{f(x)}{g(x)} = \lim_{\substack{x \rightarrow a \\ x \rightarrow \infty}} \frac{f'(x)}{g'(x)}$$

In-Class Discussion

6.8c

$$\lim_{t \rightarrow 0} \frac{8^t - 5^t}{t}$$

Check: $\frac{1-1}{0} \rightarrow \frac{0}{0} \checkmark$

$$\stackrel{H}{=} \lim_{t \rightarrow 0} \frac{\ln 8 \cdot 8^t - \ln 5 \cdot 5^t}{1} = \ln 8 - \ln 5 = \ln \frac{8}{5}$$

$\rightarrow \frac{0}{0} \checkmark$

Note: $\lim_{x \rightarrow 0} \frac{x^2 + x}{1 - e^x} \stackrel{L'H}{=} \lim_{x \rightarrow 0} \frac{2x + 1}{-e^x} = -1$

$$\lim_{x \rightarrow a^+} \frac{\cos x \ln(x - a)}{\ln(e^x - e^a)}$$

6.8f

$$\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$$

6.8g

Proof:

l'Hôpital's Rule

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that

$$\lim_{x \rightarrow a} f(x) = 0 \text{ and } \lim_{x \rightarrow a} g(x) = 0$$

$$\text{or that } \lim_{x \rightarrow a} f(x) = \pm\infty \text{ and } \lim_{x \rightarrow a} g(x) = \pm\infty$$

Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ if the limit on the right-hand side exists or is ∞ or $-\infty$

Indeterminate forms

$$\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, \infty - \infty, 0^0, \infty^0, \text{ and } 1^\infty$$



This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

In-Class Practice

Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

$$1. \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 2x}$$

$$2. \lim_{u \rightarrow \infty} \frac{e^{\frac{u}{10}}}{u^3}$$

$$3. \lim_{x \rightarrow 0} \frac{x - \sin x}{x - \tan x}$$

$$4. \lim_{x \rightarrow \infty} x^{\frac{3}{2}} \sin \left(\frac{1}{x} \right)$$

5. $\lim_{x \rightarrow 0^+} (\tan 2x)^x$