

1.7 – Diagonal, Triangular, and Symmetric Matrices

IS $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ diagonal? yes

Definition: A square matrix in which all the entries off the main diagonal are zero is called a **diagonal matrix**.

#3 Find the product by inspection.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 4 & -1 \\ 4 & 10 \end{bmatrix}$$

#8 Find A^2 , A^{-2} , and A^k (where k is any integer) by inspection.

$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

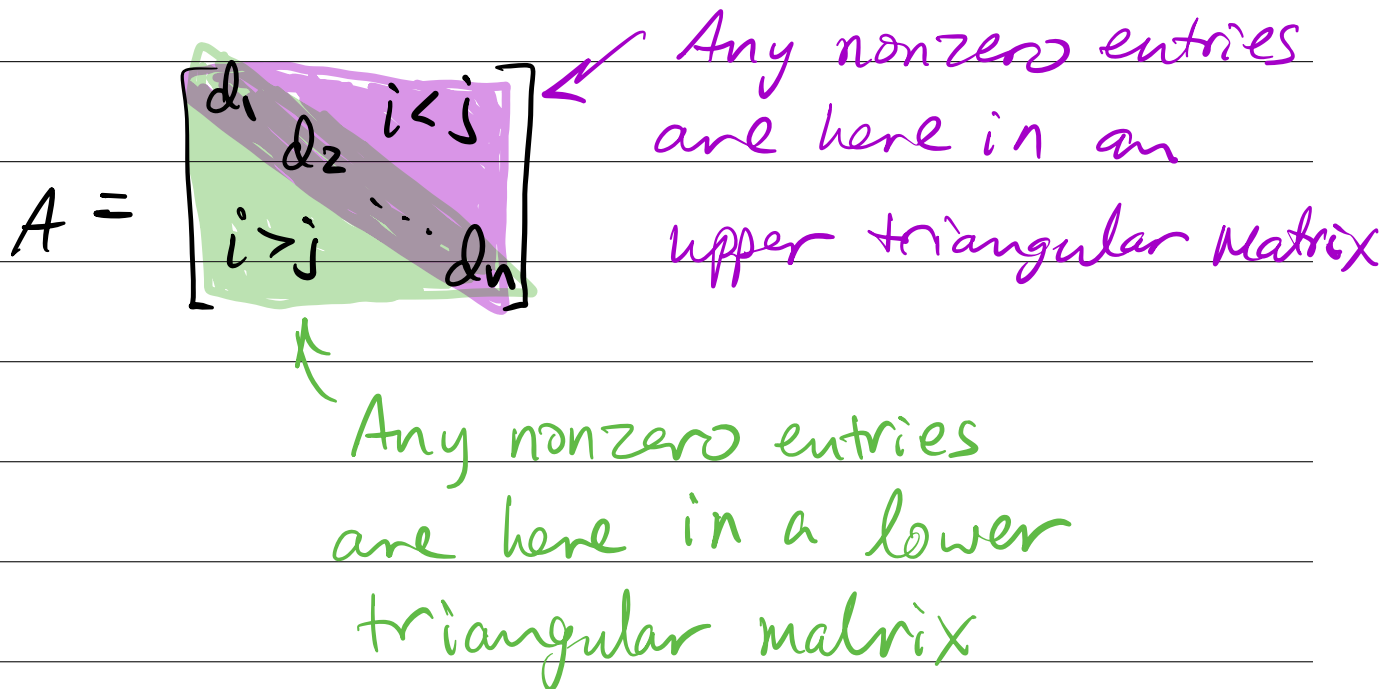
$$A^2 = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} \frac{1}{36} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{25} \end{bmatrix}$$

$$A^k = \begin{bmatrix} (-6)^k & 0 & 0 \\ 0 & 3^k & 0 \\ 0 & 0 & 5^k \end{bmatrix}$$

Definitions: A square matrix in which all entries below the main diagonal are zero is an **upper triangular** matrix. A square matrix in which all entries above the main diagonal are zero is a **lower triangular** matrix. If a matrix is either upper triangular or lower triangular (or both), it is said to be **triangular**.

Consider $A = [a_{ij}]$, a square matrix.



Theorem 1.7.1 Properties of Triangular Matrices

- a) The transpose of a lower triangular matrix is upper triangular, and the transpose of an upper triangular matrix is lower triangular.
- b) The product of lower triangular matrices is lower triangular, and the product of upper triangular matrices is upper triangular.
- c) A triangular matrix is invertible if and only if its diagonal entries are all nonzero.
- d) The inverse of an invertible lower triangular matrix is lower triangular, and the inverse of an invertible upper triangular matrix is upper triangular.

~~pf:~~ a) (lower triangular)

Let $A = [a_{ij}]$ be a lower triangular matrix. Then $a_{ij} = 0$ if $i < j$. That is, if the row number is less than the column number. But $A^T = [a_{ij}]^T = [a_{ji}]$, and $a_{ji} = 0$ for $j > i$, that is, if the row number

is greater than the column number.

A^T is then upper triangular.

#19 Determine by inspection whether the matrix is invertible.

$$\begin{bmatrix} 0 & 6 & -1 \\ 0 & 7 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

#23 Find the diagonal entries of AB by inspection.

$$A = \begin{bmatrix} 3 & 2 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 7 \\ 0 & 5 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

#46 Prove: If the matrices A and B are both upper triangular or both lower triangular, then the diagonal entries of both AB and BA are the products of the diagonal entries of A and B .

Definition: A square matrix A is said to be **symmetric** if $A = A^T$.

Theorem 1.7.2 Algebraic Properties of Symmetric Matrices

If A and B are symmetric matrices with the same size, and if k is any scalar, then:

- a) A^T is symmetric.
- b) $A + B$ and $A - B$ are symmetric.
- c) kA is symmetric.

Theorem 1.7.3 The product of two symmetric matrices is symmetric if and only if the matrices commute.

Theorem 1.7.4 If A is an invertible symmetric matrix, then A^{-1} is symmetric.

Theorem 1.7.5 If A is an invertible matrix, then AA^T and $A^T A$ are also invertible.
